The growth of plants

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Is there a way we can simulate the growth of plants using mathematics?

This is the question that I will try to answer in this presentation and the one my research topic is based on.

Fractals

By searching for informations regarding the growth of plants from a mathematical point of view, I found that it was already experimented with using fractals or Lindenmayer systems.

Fractals are complex geometric shapes that repeat at different scales and can often be observed in nature. For example, the structure of a fern leaf or the branching of trees reflect fractal patterns.

Lindenmayer systems (L-systems)

Lindenmayer systems can be a good way to represent the evolution of plants, because it does not take into account the randomness that can be found in nature, so it will be easier to find some patterns.

In the image on the left I evidentiated the simplified shape of the plant, the one that would be taken into consideration for a L-system

The difference between L-systems and fractals

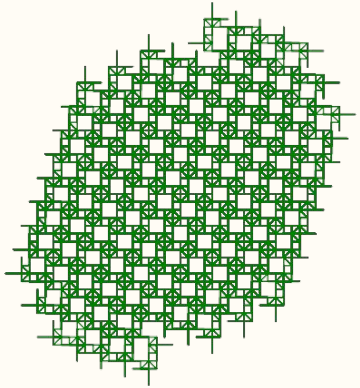
L-systems are generated by rewriting symbols based on rules, whereas fractals are typically generated by applying mathematical functions iteratively.
L-Systems excel in capturing natural forms with hierarchical structures. Fractals, on the other hand, are mathematical sets with self-similar patterns at varying scales, often described using equations or iterative algorithms.
Both have applications in nature modeling, but their approaches and underlying principles differ.
Because Lindenmayer systems were created specifically for modeling growth processes, I chose to continue the research using them.

What are L-systems?

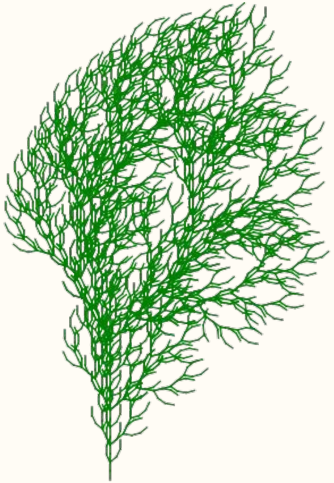
Lindenmayer systems are a type of formal language primarily used to simulate a simplified version of the growth of plants. These types of systems were introduced by Aristid Lindenmayer, a Hungarian biologist, in 1968.
The fundamental components for this type of systems are the following:

- The alphabet, which is made of a number of characters, each of them having a specific meaning, which are used to create strings
- The rule that has the purpose of determining what characters get replaced and with what string when we go from a generation to another
- The axiom, that represents the string we are starting with

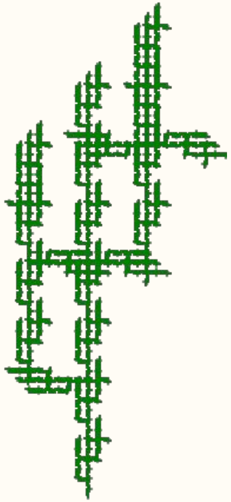
Exemples of representations using L-systems



Rule :
F=FF[+F-F--F]F---F[++F--FFF]



Rule:
F=FF-[-F+F+F]+[+F-F-F]
(25 degrees angle)



Rule:
F=FF[+F-F[F-F+F][FFF-
-F++]]FF[-F+F[F+F-F][
FFF--FF+F]]FF

These images were created with python turtle graphics

Let’s focus on an example.

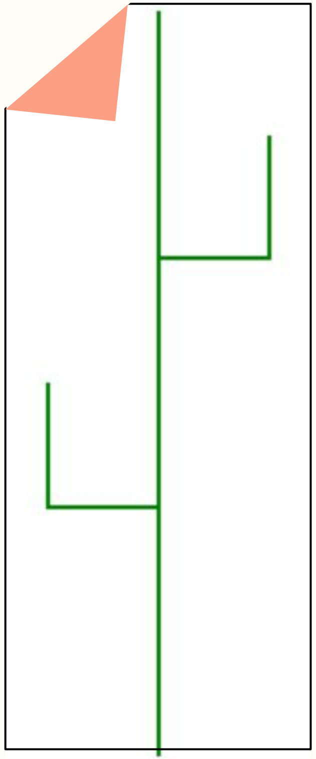
The cactus

We define an alphabet:

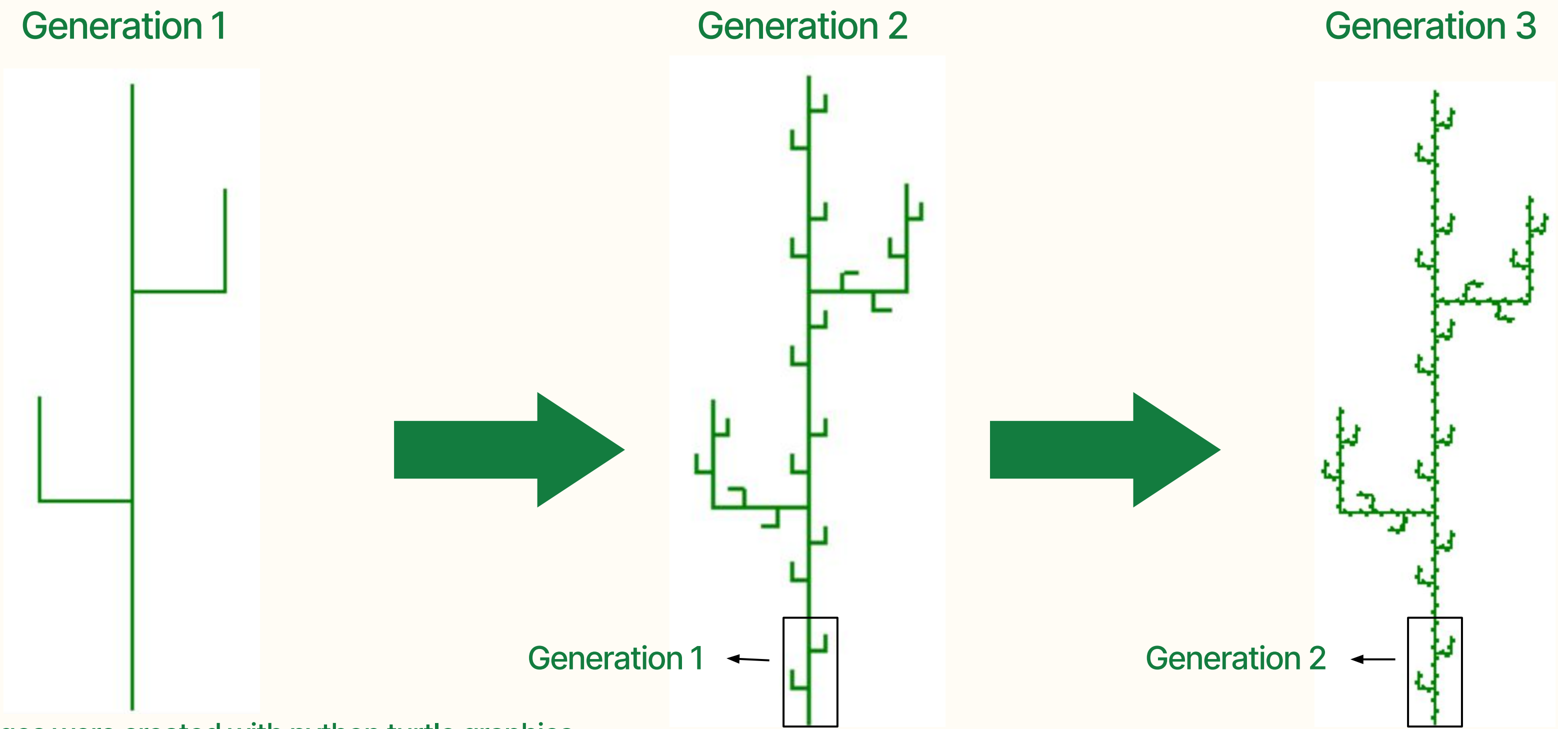
F (take one step forward)
+ (turn left 90°)
- (turn right 90°)
[(Remember the position)
] (Return to last remembered position)

And a rule:
F=FF[+F-F]FF[-F+F]FF
Our axiom is F.

Generation 1



This is how the generations of the cactus would look like



These images were created with python turtle graphics

What changes from a generation to another in the case of the cactus?

Number of generation	Height	Number of F's	*Number of [,], +, -
1	6	10	2
2	36	100	22
3	216	1000	222

*The number of [,], +, - (it's the same number for each of these symbols and it also represents the number of branches)

By observing these changes we could say the following: (n is the number of the generation)

Height=6^n

Number of F's=10^n

Number of [,], +, -=$2*(10^0+10^1+10^2+...+10^n)$

We can find a shorter formula for the number of [,], +, -

$$\begin{array}{ccccccc} 22\ldots2 = 2 \times (& 1 & + & 10 & + & 100 + \ldots + & 10^{n-1}) \\ & \| & & \| & & \| & \| \\ & b_1 & & b_2 & & b_3 & b_n \end{array}$$

$$\frac{b_n}{b_{n-1}} \stackrel{?}{=} constant \qquad \frac{10^{n-1}}{10^{n-2}} = \frac{10^{n-2} \times 10}{10^{n-2}} = 10 = constant$$

$=> 1, 10, 100, \ldots 10^{n-1}$ are part of a geometric sequence

$$1 + 10 + 100 + \ldots + 10^{n-1} = \frac{10^n - 1}{9}$$

$$=> 22\ldots2 = 2 \times \left(\frac{10^n - 1}{9} \right)$$

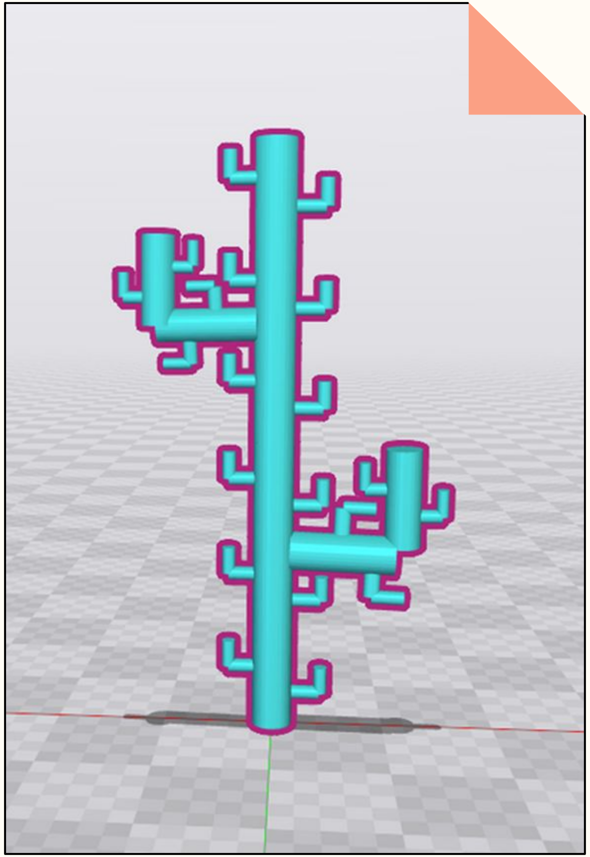
Now, we could organize the formulas in a table

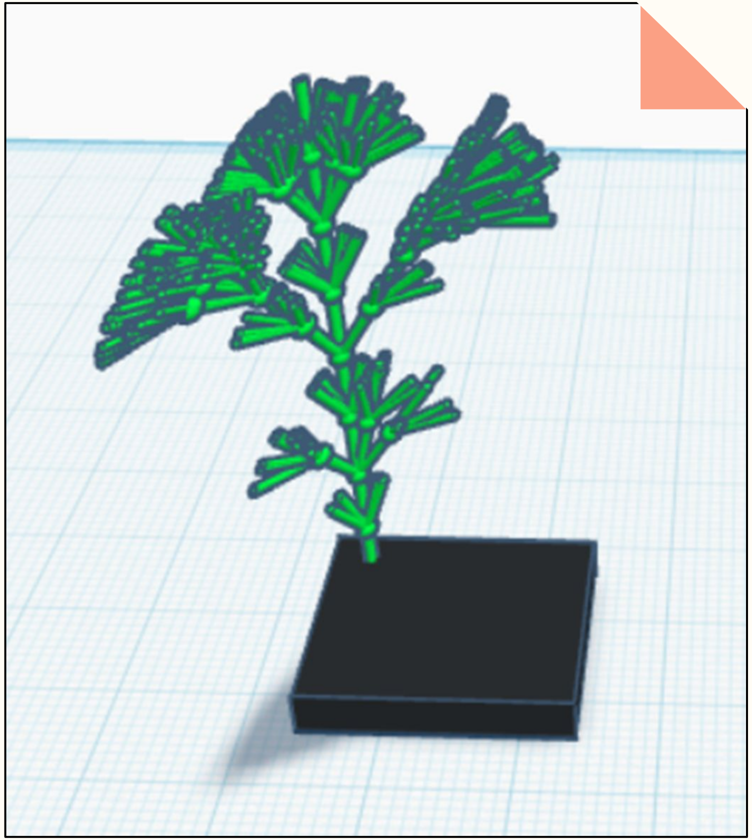
Number of generation	Height (if F=1 unit)	Number of F's	Number of +, -, [,]
1	6	10	2
2	36	100	22
3	216	1000	222
⋮	⋮	⋮	⋮
n	6^n	10^n	$2\frac{10^n - 1}{9}$

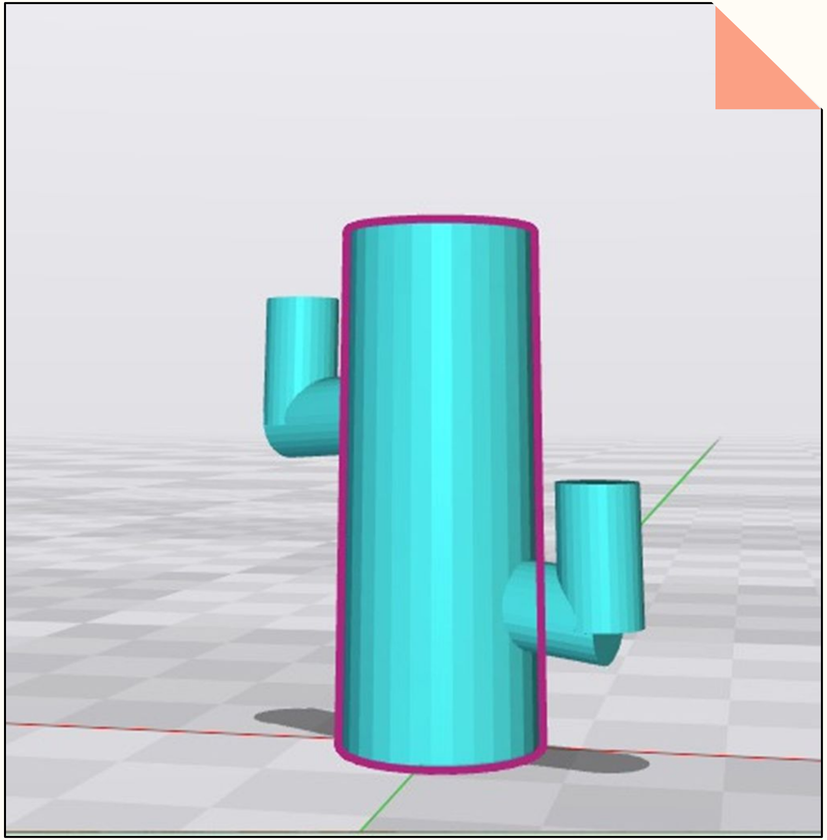
Physical representation

Because of the direct connection of the research problem with Fractals and L-System, I used a 3D printer to bring the resulted patterns from the digital world into the real world.

Fractals, known for their self-similar and intricate structures, can be translated into tangible objects using 3D modeling software. These prints reveal stunning geometric designs, often mimicking natural forms like trees, snowflakes or coral.







Conclusion

Mathematics can be found in the evolution of plants but we need to analyze simplified versions to be able to find formulas. It is also necessary to observe every plant individually because the rules each one follows are different for each one.

To achieve this, Lindenmayer systems are useful, helping with understanding the evolution of plants.