MATHEMATICAL EXPERIMENT COMPETITION

MAJA NOWAK

Optimal Gear Ratios in Cycling

Context and Research Question

As a person who enjoys cycling and cycles frequently for leisure, I often encounter different terrains along my path. Therefore, I turn the gears in hopes of finding a suitable combination depending on the terrain, as each terrain requires a different combination. On a typical mountain bike with three chainrings at the front and six chainrings at the back, I am provided with eighteen different combinations, making finding the appropraite combination difficult.

Thus I formulated my **research question**:

'**What is the optimal gears ratios for riding a bike on different terrains: flat, uphill and downhill?**'

Hypotheses

Hypothesis for cycling on a flat road: The fastest gear ratio for cycliing on a flat road will be between 2.000 and 3.000

Explanation: A gear ratio of between 2.000 and 3.000 means that for every full rotation of the cranks, the wheel will spin over twice its' circumference providing a moderate comfortable pace

Hypothesis for cycling uphill: The fastest gear ratio for cyclying uphill will be between 3.000 and 4.000

Explanation: Despite more force being required, a high gear ratio will mean you cycle further for every full rotation of the cranks, meaning there is less exertion on the legs.

Hypothesis for cycling downhill: The fastest gear ratio for cycling downhill will be between 1.000 and 2.000

Explanation: To continue to cycle downhill whilst maintaining control, a moderately low gear ratio will suffice.

Background Information

What are gear ratios?

A gear ratio on a bike tells us how many times the rear wheel turns in one rotation of the cranks. It is the quotient of the number of teeth on the front chainring divided by the number of teeth on the back chainring.

$$\textit{Gear Ratio} = \frac{\textit{Number of teeth on the front chainring}}{\textit{Number of teeth on the back chainring}}$$

The challenges for each terrain

The terrains on which I will be focusing on are; flat, uphill and downhill. These are the most common terrains found by all cyclists hence why I will be using these for comparisons.

Challenges

Flat: To maintain speed

Uphill: To maximise speed without too much fatigue

Downhil: To maximise speed and maintain control

Method

I decided to approach my research question by timing a person riding a bike for a fixed distance to calculate their average speed. I believe that this angle is appropraite as you generally want to maximise speed with comfort. The participant was asked to cycle at a comfortable speed without any overexertion and to peddle at approximately the same pace for the 50 meters.

1. Count the teeth on each of the front and back chainrings on the bike and create a table for calculating the gear ratio for each combination using the formula provided.
2. Measure a fixed distance of 50m on a flat road and on a hill using a long measuring tape.
3. Time a person cycling the 50m on a flat road for each gear combination using a stopwatch, and repeat three times for each to calculate the average time by adding the three times and dividing by 3.
4. Use the average time to calculate the average speed using: $Speed = \frac{Distance}{Time}$
5. Repeat for uphill and downhill for 50m.

Photos taken during the experiment

Hill

Flat road

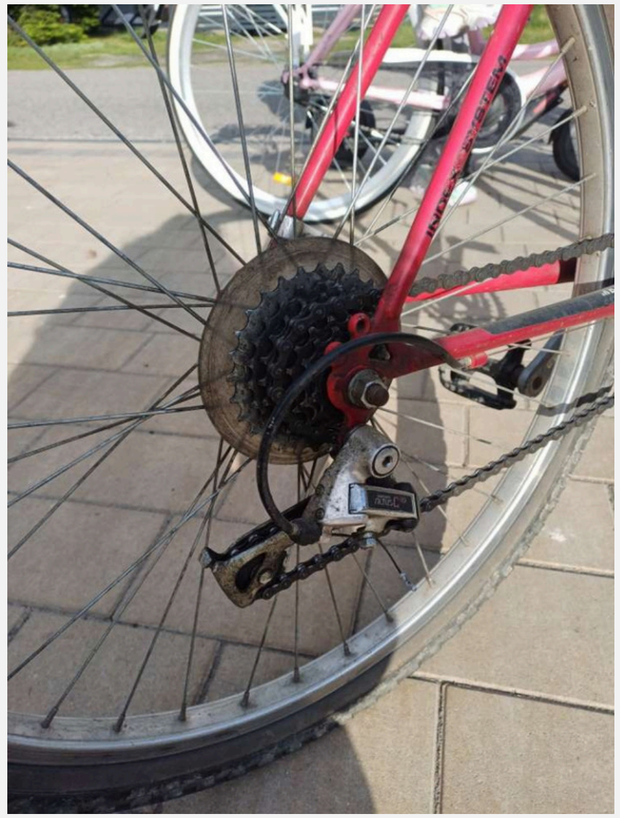
Raw Data

Number of teeth on each chainring

Number of Teeth on Front Chainring	
Gear 1):	48
Gear 2):	38
Gear 3):	28

Raw Data

Number of teeth on each chainring



Number of Teeth on Back Chainring	
Gear 1):	28
Gear 2):	24
Gear 3):	22
Gear 4):	18
Gear 5):	16
Gear 6):	14

Raw Data

Calculated Gear Ratios for each combination

		Back Chainring					
		Gear 1	Gear 2	Gear 3	Gear 4	Gear 5	Gear 6
Front Chainring	Gear 1	1.714	2.000	2.182	2.667	3.000	3.429
	Gear 2	1.357	1.583	1.727	2.111	2.375	2.714
	Gear 3	1.000	1.167	1.273	1.556	1.750	2.000

Raw Data

Times for flat road, and average time

Gear combination	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Time 1 (s)	15.80	16.94	14.64	15.53	13.22	14.13	18.70	17.02	17.43	15.20	15.09	14.06	23.53	21.74	21.90	19.40	17.42	16.39
Time 2 (s)	17.15	14.99	15.43	13.27	13.96	13.07	17.83	17.97	16.31	16.43	14.94	14.97	24.98	22.93	20.22	17.96	17.84	16.44
Time 3 (s)	16.83	15.54	14.94	13.66	13.65	14.09	19.14	16.58	16.98	15.37	15.64	13.73	25.79	23.31	20.03	18.22	16.98	16.75
Average Time (s)	16.59	15.82	15.00	14.15	13.61	13.76	18.56	17.19	16.91	15.67	15.22	14.25	24.77	22.66	20.72	18.53	17.41	16.53

Raw Data

Times for uphill, and average time

Gear combination	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Time 1 (s)	19.21	17.65	17.39	16.74	17.01	16.05	22.95	22.23	20.42	18.32	17.45	17.05	24.72	21.24	19.16	18.58	17.30	15.43
Time 2 (s)	18.79	17.18	16.25	16.46	16.13	15.34	22.89	20.90	19.30	17.57	16.78	16.46	23.24	21.76	19.38	16.79	17.04	16.81
Time 3 (s)	19.56	18.22	16.89	17.12	16.39	15.68	23.11	21.82	21.05	19.09	17.52	16.23	22.68	22.12	20.54	19.02	18.50	15.98
Average Time (s)	19.19	17.68	16.84	16.77	16.51	15.69	22.98	21.65	20.26	18.33	17.25	16.58	23.55	21.71	19.69	18.13	17.61	16.07

Raw Data

Times for downhill, and average time

	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Time 1	14.32	13.33	12.67	12.15	12.20	12.09	15.39	15.89	14.84	12.84	11.93	11.75	16.18	15.45	13.73	13.41	11.81	11.33
Time 2	13.87	13.01	12.61	12.41	12.22	11.41	15.03	14.77	13.92	11.74	12.10	12.27	16.65	15.38	13.95	13.15	12.93	11.62
Time 3	14.22	13.92	13.06	12.81	12.07	11.64	15.76	15.09	13.89	12.01	11.53	11.67	16.79	15.22	14.21	13.57	12.51	11.83
Average Time	14.14	13.42	12.78	12.46	12.16	11.71	15.39	15.25	14.22	12.20	11.85	11.90	16.54	15.35	13.96	13.37	12.42	11.59

Data

Average speed for flat road gear combinations

Gear combination	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Speed (m/s)	3.01	3.16	3.33	3.53	3.67	3.63	2.69	2.91	2.96	3.19	3.29	3.51	2.02	2.21	2.41	2.70	2.87	3.02

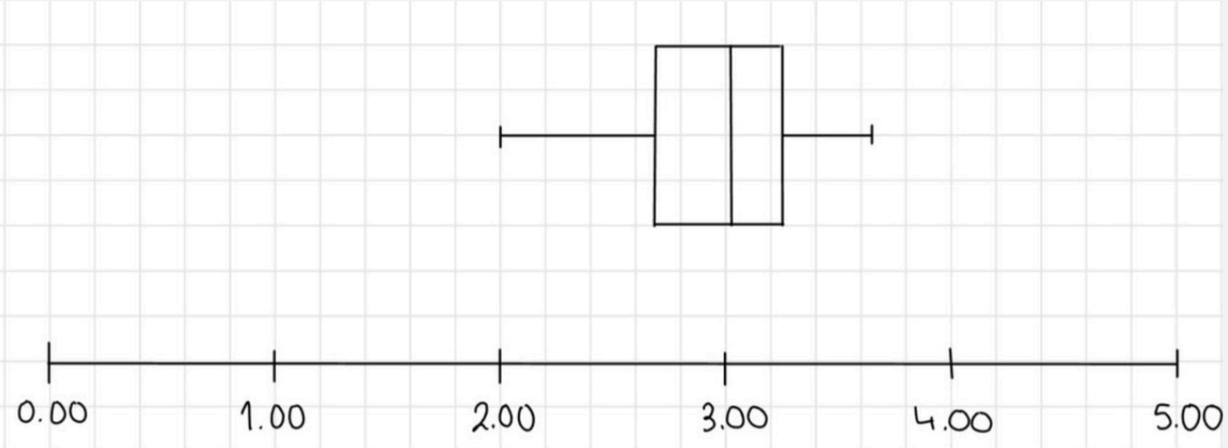
Maximum Value: 3.67

Minimum Value: 2.02

Median: 3.015

Upper Quartile: 3.33

Lower Quartile: 2.70



Data

Average speed for uphill gear combinations

Gear combination	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Speed (m/s)	2.61	2.83	2.97	2.98	3.03	3.19	2.18	2.31	2.48	2.73	2.90	3.02	2.12	2.30	2.54	2.76	2.84	3.11

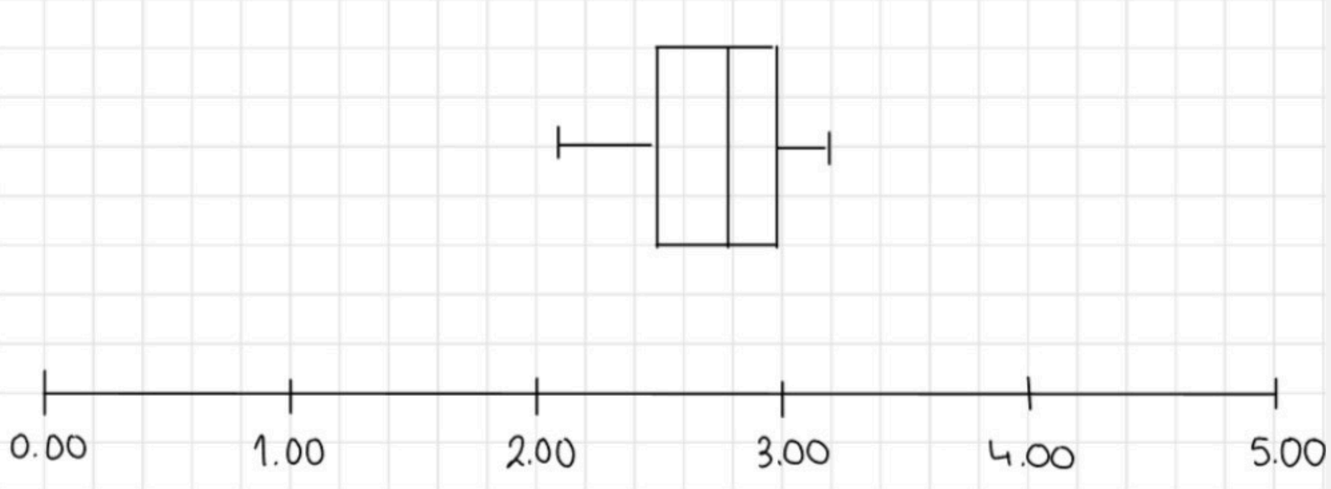
Maximum Value: 3.19

Minimum Value 2.12

Median: 2.795

Upper Quartile: 2.98

Lower Quartile: 2.48



Data

Average speed for downhill gear combinations

Gear combination	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6
Speed (m/s)	3.54	3.73	3.91	4.01	4.11	4.27	3.25	3.28	3.52	4.10	4.22	4.20	3.02	3.26	3.58	3.74	4.03	4.31

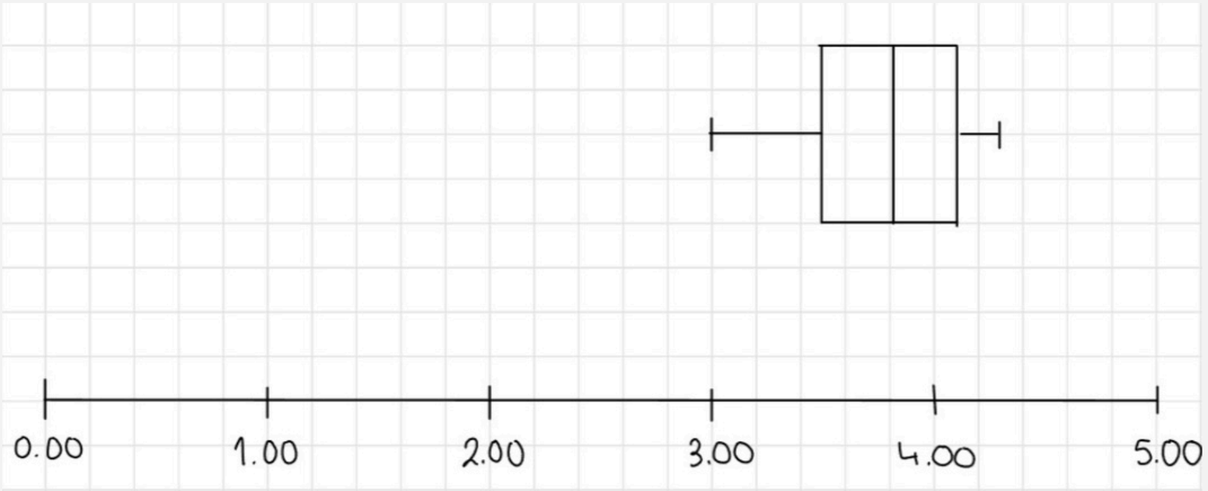
Maximum Value: 4.31

Minimum Value: 3.02

Median: 3.825

Upper Quartile: 4.11

Lower Quartile: 3.52



Conclusion

After conducting my experiment, I am in the position to accept or reject my hypotheses and suggest explanations.

Hypothesis for cycling on a flat road: My hypothesis was correct. The fastest gear ratio was 3.000, a combination of Gear 1 on the frontring and Gear 5 on the back chainring.

Hypothesis for cycling uphill: My hypothesis was correct. The fastest gear ratio resulted to be 3.429, a combination of Gear 1 on the front chainring and Gear 6 on the back chainring.

Hypothesis for cycling downhill: My hypothesis was correct. The fastest gear ratio resulted to be 2.000, a combination of Gear 3 on the front chainring and Gear 6 on the back chainring.

Evaluations

There are possible improvements or extensions for my experiment.

Improvements

1. Distance: The distance may have been too small for the cyclist to feel overexerted at any point, allowing them to cycle faster on higher gear ratios. A larger distance may have improved my findings and correlate more closely with my hypotheses.
2. Weather conditions: To reduce impact of weather conditions, the experiment was done over the course of 2 consecutive days. However, there was a slight breeze from time to time which may have impacted the speeds. To improve this, I could aim to complete the experiment in a single day.
3. Peddling: The participant was told to peddle at a "comfortable pace", which may be different for many. As an improvement, I could record and incorporate cadence into my experiment.

Extensions

1. The experiment could be repeated for more terrains, such as on gravel and grass. This provides more data for analysis and interpretation.
2. The experiment could employ more participants. This covers for different fitness levels which vary for people, making my experiment more reliable.

Thank You!

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